

Mathematica 11.3 Integration Test Results

Test results for the 175 problems in "Apostol Problems.m"

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{t^3}{\sqrt{4+t^3}} dt$$

Optimal (type 4, 172 leaves, 2 steps):

$$\frac{2}{5} t \sqrt{4+t^3} - \frac{8 \times 2^{2/3} \sqrt{2+\sqrt{3}} (2^{2/3}+t) \sqrt{\frac{2 \cdot 2^{1/3} - 2^{2/3} t + t^2}{(2^{2/3}(1+\sqrt{3})+t)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{2/3}(1-\sqrt{3})+t}{2^{2/3}(1+\sqrt{3})+t}\right], -7-4\sqrt{3}\right]}{5 \times 3^{1/4} \sqrt{\frac{2^{2/3}+t}{(2^{2/3}(1+\sqrt{3})+t)^2}} \sqrt{4+t^3}}$$

Result (type 4, 122 leaves):

$$\frac{6 t (4+t^3) - 8 (-2)^{1/6} 3^{3/4} \sqrt{-(-1)^{1/6} (2(-1)^{2/3} + 2^{1/3} t)} \sqrt{4+2(-2)^{1/3} t + (-2)^{2/3} t^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-i+\sqrt{3})(2+2^{1/3} t)}}{2 \cdot 3^{1/4}}\right], (-1)^{1/3}\right]}{15 \sqrt{4+t^3}}$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int x^4 (1+x^5)^5 dx$$

Optimal (type 1, 11 leaves, 1 step):

$$\frac{1}{30} (1+x^5)^6$$

Result (type 1, 43 leaves):

$$\frac{x^5}{5} + \frac{x^{10}}{2} + \frac{2x^{15}}{3} + \frac{x^{20}}{2} + \frac{x^{25}}{5} + \frac{x^{30}}{30}$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int (1-x)^{20} x^4 dx$$

Optimal (type 1, 56 leaves, 2 steps):

$$-\frac{1}{21} (1-x)^{21} + \frac{2}{11} (1-x)^{22} - \frac{6}{23} (1-x)^{23} + \frac{1}{6} (1-x)^{24} - \frac{1}{25} (1-x)^{25}$$

Result (type 1, 140 leaves):

$$\frac{x^5}{5} - \frac{10x^6}{3} + \frac{190x^7}{7} - \frac{285x^8}{2} + \frac{1615x^9}{3} - \frac{7752x^{10}}{5} + \frac{38760x^{11}}{11} - 6460x^{12} + 9690x^{13} - \frac{83980x^{14}}{7} + \frac{184756x^{15}}{15} - \frac{20995x^{16}}{2} + 7410x^{17} - \frac{12920x^{18}}{3} + 2040x^{19} - \frac{3876x^{20}}{5} + \frac{1615x^{21}}{7} - \frac{570x^{22}}{11} + \frac{190x^{23}}{23} - \frac{5x^{24}}{6} + \frac{x^{25}}{25}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+3\cos[x]^2} \sin[2x] dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$-\frac{2}{9} (4-3\sin[x]^2)^{3/2}$$

Result (type 3, 49 leaves):

$$\frac{5\sqrt{5} - 5\sqrt{5+3\cos[2x]} - 3\cos[2x]\sqrt{5+3\cos[2x]}}{9\sqrt{2}}$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int \text{ArcSec}[x] dx$$

Optimal (type 3, 19 leaves, 4 steps):

$$x \text{ArcSec}[x] - \text{ArcTanh}\left[\sqrt{1-\frac{1}{x^2}}\right]$$

Result (type 3, 64 leaves):

$$x \operatorname{ArcSec}[x] - \frac{\sqrt{-1+x^2} \left(-\operatorname{Log}\left[1 - \frac{x}{\sqrt{-1+x^2}}\right] + \operatorname{Log}\left[1 + \frac{x}{\sqrt{-1+x^2}}\right] \right)}{2 \sqrt{1 - \frac{1}{x^2}} x}$$

Problem 84: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCsc}[x] \, dx$$

Optimal (type 3, 17 leaves, 4 steps):

$$x \operatorname{ArcCsc}[x] + \operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{x^2}}\right]$$

Result (type 3, 64 leaves):

$$x \operatorname{ArcCsc}[x] + \frac{\sqrt{-1+x^2} \left(-\operatorname{Log}\left[1 - \frac{x}{\sqrt{-1+x^2}}\right] + \operatorname{Log}\left[1 + \frac{x}{\sqrt{-1+x^2}}\right] \right)}{2 \sqrt{1 - \frac{1}{x^2}} x}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\cos[x] + \sin[x]} \, dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\cos[x] - \sin[x]}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 24 leaves):

$$(-1 - i) (-1)^{3/4} \operatorname{ArcTanh}\left[\frac{-1 + \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right]$$

Problem 154: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x+x^2}} \, dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{x+x^2}}\right]$$

Result (type 3, 29 leaves):

$$\frac{2 \sqrt{x} \sqrt{1+x} \operatorname{ArcSinh}[\sqrt{x}]}{\sqrt{x(1+x)}}$$

Problem 175: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{1+t^3}} dt$$

Optimal (type 4, 103 leaves, 1 step):

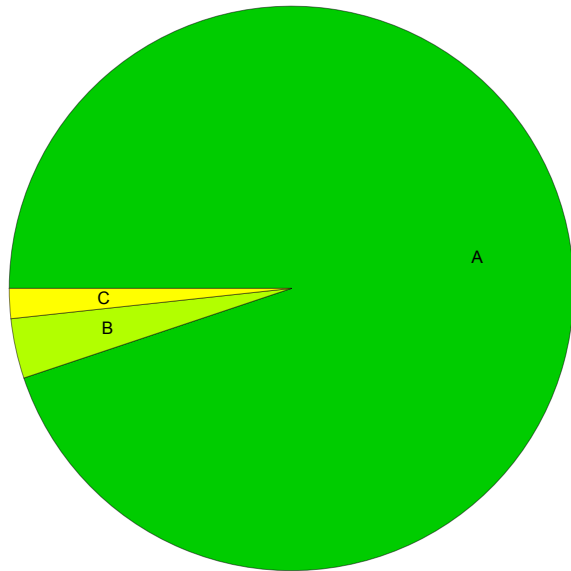
$$\frac{2 \sqrt{2+\sqrt{3}} (1+t) \sqrt{\frac{1-t+t^2}{(1+\sqrt{3}+t)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+t}{(1+\sqrt{3}+t)^2}} \sqrt{1+t^3}}$$

Result (type 4, 88 leaves):

$$\frac{1}{3^{1/4} \sqrt{1+t^3}} 2 (-1)^{1/6} \sqrt{-(-1)^{1/6} ((-1)^{2/3} + t)} \sqrt{1 + (-1)^{1/3} t + (-1)^{2/3} t^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+t)}}{3^{1/4}}\right], (-1)^{1/3}\right]$$

Summary of Integration Test Results

175 integration problems



A - 166 optimal antiderivatives

B - 6 more than twice size of optimal antiderivatives

C - 3 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts